More 5.3

Properties of Definite Integrals

① $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

② $\int_a^a f(x)\,dx = 0$ ③ $\int_a^b K\cdot f(x)\,dx = K\int_a^b f(x)\,dx$

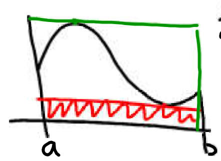
④ $\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

⑤ $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

⑥ Max/Min Inequality:

If maxf and minf are max & min of f on [a,b]

then $\text{minf}\,(b-a) \le \int_a^b f(x)\,dx \le \text{maxf}\,(b-a)$



⑦ Domination:

If $f(x) \ge g(x)$ on $[a,b]$

then $\int_a^b f(x)\,dx \ge \int_a^b g(x)\,dx$

If: $\int_{-1}^{1} f(x)\,dx = 5$ and $\int_1^4 f(x)\,dx = -2$ and $\int_{-1}^{1} h(x)\,dx = 7$

① $\int_4^1 f(x)\,dx = 2$ ② $\int_{-1}^{4} f(x)\,dx = 5 + -2 = 3$

③ $\int_{-1}^{1} [\boxed{2}\, f(x) + 3\,h(x)]\,dx$

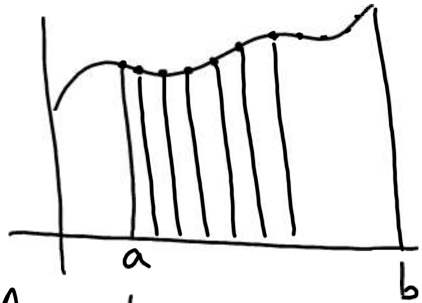
$= 2\int_{-1}^{1} f(x)\,dx + 3\int_{-1}^{1} h(x)\,dx = 31$

$= 2(5) + 3(7) = 31$

④ $\int_0^1 f(x)\,dx$ = not enough information

⑤ $\int_{-2}^{2} h(x)\,dx$ = not enough information

Average Value of a Function



$\Delta x = \frac{b-a}{n}$

$\frac{\Delta x}{b-a} = \frac{1}{n}$

average value:

$$\frac{f(c_1)+f(c_2)+f(c_3)+\dots+f(c_n)}{n}$$

$$= \frac{1}{n} \cdot \sum_{k=1}^{n} f(c_k)$$

$$= \frac{\Delta x}{b-a} \cdot \sum_{k=1}^{n} f(c_k)$$

$$= \frac{1}{b-a} \cdot \boxed{\sum_{k=1}^{n} f(c_k) \cdot \Delta x}$$

$$= \frac{1}{b-a} \int_a^b f(x)\,dx$$

If f is integrable on $[a,b]$ its mean value on $[a,b]$ is

$$av(f) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

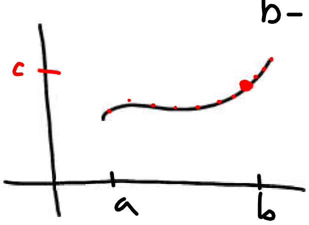
TRY: Find $av(f)$ for $f(x) = 4-x^2$ on $[0,3]$.

$$av(f) = \frac{1}{3-0}\int_0^3 (4-x^2)\,dx = \frac{1}{3}(3) = 1$$

Mean Value Theorem for Integrals

If $f(x)$ is continuous on $[a,b]$ then at some point c in $[a,b]$

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$



Fundamental Theorem of Calculus

If $F(x)$ is any antiderivative of $f(x)$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

① $$\int_0^{\pi} \sin x\,dx = F(\pi) - F(0)$$

$F(x) = -\cos x$

$$= -\cos\pi - (-\cos 0)$$
$$= -(-1) - (-1)$$
$$= 2$$

② $$\int_3^7 8\,dx = F(7) - F(3)$$

$F(x) = 8x$

$$= 8(7) - 8(3)$$
$$= 32$$

③ $$\int_0^{1/2} \frac{1}{\sqrt{1-x^2}}\,dx = F(\tfrac{1}{2}) - F(0)$$

$F(x) = \sin^{-1}x$

$$= \sin^{-1}(\tfrac{1}{2}) - \sin^{-1}(0)$$
$$= \frac{\pi}{6} - 0$$
$$= \frac{\pi}{6}$$